A Guide for Designing with Radial Gradient-Index Materials Constrained to Real Material Properties in Code V

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I. Material Model

The gradient-index material model assumes a smooth change in composition from one homogeneous material to another. Any two-component GRIN system can be modeled with these tools. The refractive index is represented as

$$n = n_1 + C_2(n_2 - n_1),$$

where n_1 is the refractive index of material one and C_2 is the fractional concentration of material two (ranging from zero to one). This linear representation is not strictly true for all materials, but it is a useful approximation for modeling the material properties.

The refractive index distribution is given the usual polynomial expansion representation

$$n(r) = N_{00} + N_{10}r^2 + N_{20}r^4 + N_{30}r^6 + \cdots.$$

One can also consider the composition profile of the material

$$C(r) = C_{00} + C_{10}r^2 + C_{20}r^4 + C_{30}r^6 + \cdots.$$

For a given composition profile, the refractive index profile is fixed for all wavelengths based on the linear model of index vs. composition. However, the standard implementation of gradient-index materials in Code V uses the index function representation. As such, it is essential for the designer to ensure the index profile at different wavelengths agrees with the real material properties. The following sections of this document describe a set of tools to accomplish this task.

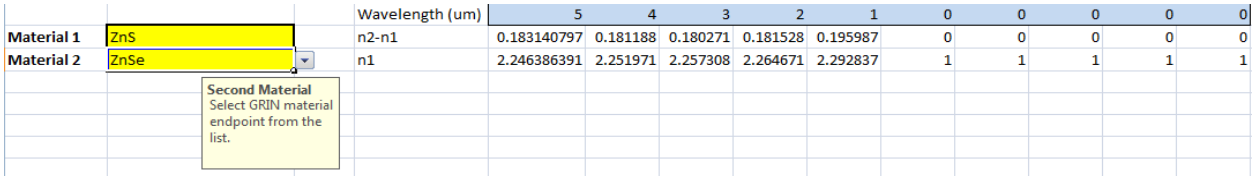
II. Entering a GRIN Profile in Code V

The bulk of the design tools are contained within one Excel spreadsheet. This file contains material properties for several possible GRIN materials as well as the constraints for optimization in Code V. The first worksheet of the Excel file, “Material Properties”, contains the material properties for several candidate GRIN materials, including ALON, ZnS/ZnSe, PMMA-Polystyrene, and a Titania-Silicate GRIN glass. For a given material the spreadsheet looks like this:

PMMA-Polystyrene		Wavelength (um)	0.656273	0.587562	0.486133	0	0	0	0	0	0	0
Index measurements in literature		n2-n1	0.097317	0.100302	0.108296	1	1	1	1	1	1	1
from 435.8 nm to 1064 nm		n1	1.487957	1.491402	1.497298	1	1	1	1	1	1	1

The cells that are highlighted in yellow indicate fields that require user input. The user can enter up to ten wavelengths (in microns, ordered from longest to shortest). The first three wavelengths are required and the remaining seven are optional. Use a value of zero for any wavelength you do not wish to use. The values for slope and y-intercept are calculated based on the material and the wavelengths chosen. New materials combinations can be added if the dispersion formulae for the two homogeneous materials are known. Keep in mind that manufacturing limitations should also be considered.

The second worksheet, "Create New Materials", includes the framework for adding new GRIN materials, as well as a way to easily select the materials for n_1 and n_2 from a drop-down menu. New homogeneous materials can be added to the available choices by simply inserting a Sellmeier equation (or equivalent dispersion fit) in the same manner that the existing materials are defined.



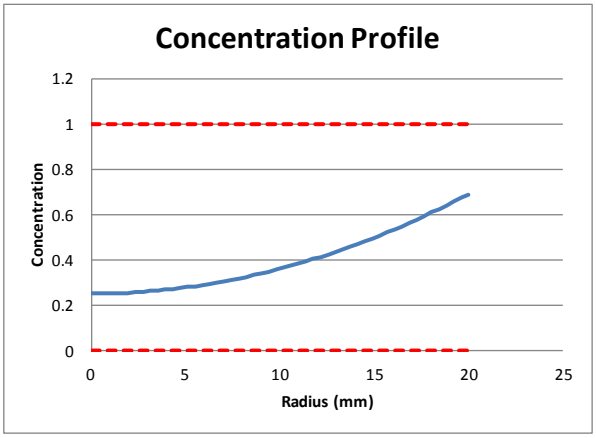
Once the desired material and wavelengths have been selected, the user should copy the wavelength, slope, and y-intercept data and paste it in the second worksheet of the file ("Radial GRIN Controls") as seen below.

Material Data:		Wavelength (um)	0.6562725	0.5875618	0.4861327	0	0	0	0	0	0	0
n = (n2-n1)*(Concentration) + n1		n2-n1	0.0973169	0.100302084	0.108296359	1	1	1	1	1	1	1
		n1	1.4879565	1.491402003	1.497297857	1	1	1	1	1	1	1

From here, the next step is to input the GRIN profile that you wish to start with. Again, cells that are highlighted and outlined in bold require user input. The controls for the starting profile include the concentration at r = 0 and the index polynomial coefficients (N_{10}, N_{20}, N_{30}) at the longest wavelength. The user also needs to enter the maximum semi-diameter of the lens (rmax) in order to ensure that the index values are staying within the material bounds.

Input initial GRIN profile			Refractive Index									
		Concentration	1.3 um	1.2 um	1.1 um	1 um	0.9 um	0.8 um	0.7 um	0.6 um	0.5 um	0.4 um
At r = 0		0.25	1.496943175	1.500379323	1.502961775	1.50479881	1.50616764	1.507662319	1.51036204	1.51561725	1.52321623	1.54904125
Maximum		0.68918595	1.536943175	1.54012267	1.5426532	1.5446678	1.54647194	1.548693636	1.55247499	1.55939067	1.57012416	1.60053018
Minimum		0.2500	1.496943175	1.500379323	1.502961775	1.50479881	1.50616764	1.507662319	1.51036204	1.51561725	1.52321623	1.54904125
		|Δn|	4.000E-02	3.974E-02	3.969E-02	3.987E-02	4.030E-02	4.103E-02	4.211E-02	4.377E-02	4.691E-02	5.149E-02
rmax	20											
		Vary Coefficient? (yes or no)										
N0		yes	1.496943175	1.500379323	1.502961775	1.50479881	1.50616764	1.507662319	1.51036204	1.51561725	1.52321623	1.54904125
N10		yes	1.0000E-04	9.9358E-05	9.9229E-05	9.9672E-05	1.0076E-04	1.0258E-04	1.0528E-04	1.0943E-04	1.1727E-04	1.2872E-04
N20		yes	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
N30		yes	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

A plot of the concentration profile is provided so the user can make sure the profile is within the material bounds (usually zero to one).



The user can also control whether the coefficients are varied during optimization in Code V by inputting “yes” or “no” in the appropriate boxes.

Below the profile controls, several first-order values are calculated based on a user-entered thickness value. These include the focal length, minimum possible focal length, and Abbe numbers of the base material and the gradient.

Calculate first-order properties based on input thickness						
Thickness	10					
EFL =	-501.12		Min possible EFL =		57.79394561	
	Based on wavelengths 1, 3, and 5			Based on wavelengths 6, 8, and 10		
V0 =	54.524783			V0 =	12.46086509	
V10 =	130.43362			V10 =	4.185795479	

Next, the user has control over the concentration bounds of the material (C_2 from section I above). For example, with a PMMA-Polystyrene gradient the physical material bounds are between zero and one. If a manufacturing process is not able to achieve that full composition change, then the designer would change those appropriately (for example, make them 0.1 to 0.9).

When using the gradient ALON, there is some flexibility here. The composition range of zero to one represents the range of composition that has been fabricated in homogeneous ALON samples. However, GRIN samples have been made with an effective composition range of -1.64 to 1.57. These values are reasonable for the designer to use. This tool allows the designer to change these limits and explore the design space, in turn motivating further development of the ALON GRIN material.

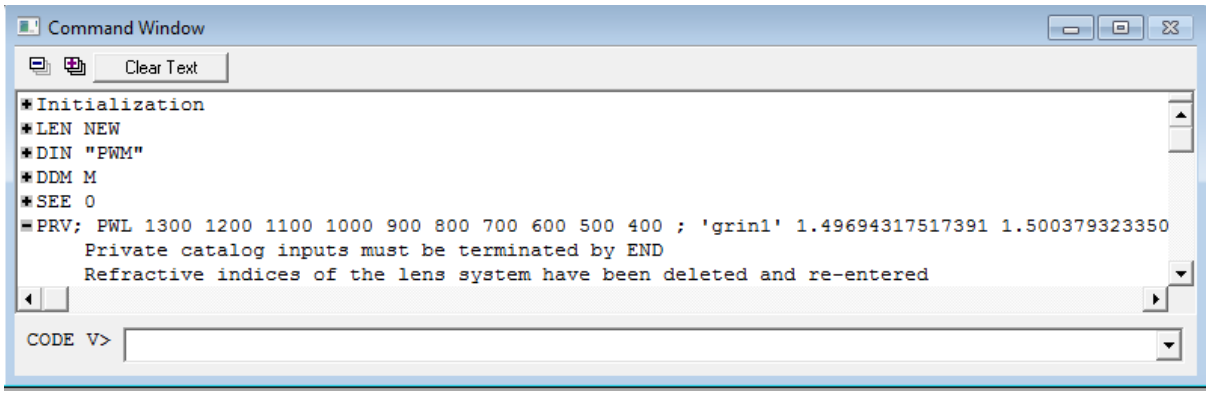
Minimum concentration		0	Constrain concentration to be within these manufacturable limits		
Maximum concentration		1	Should be (0,1) for all real materials, and ALON can safely be pushed to (-1.64,1.57)		

The last user-entered values relate to inserting this gradient profile into Code V. The “Element ID” field is used to name the private catalog glass. The ID can be a number or it can be up to four characters that describe the GRIN material (for example “pmma” or “ALON”). This same name also must be given to the surface that contains the GRIN material. For example, if the user enters “3” for the element ID then the private catalog glass is titled ‘grin3’ and the surface that contains this material must also be labeled grin3. If the element ID is “pmma” then the private catalog glass is titled ‘grinpmma’ and the surface must be labeled grinpmma. The last user input is the step size (in mm) for the ray tracing calculation. A smaller value will be more accurate but require longer processing time. The designer should ensure that the step size is small enough so it is not introducing ray trace errors. This can be checked by tracing a real ray through the lens system and monitoring how its output height changes with step size. The step size can be changed in the “Surface Properties” section of the Code V GUI.

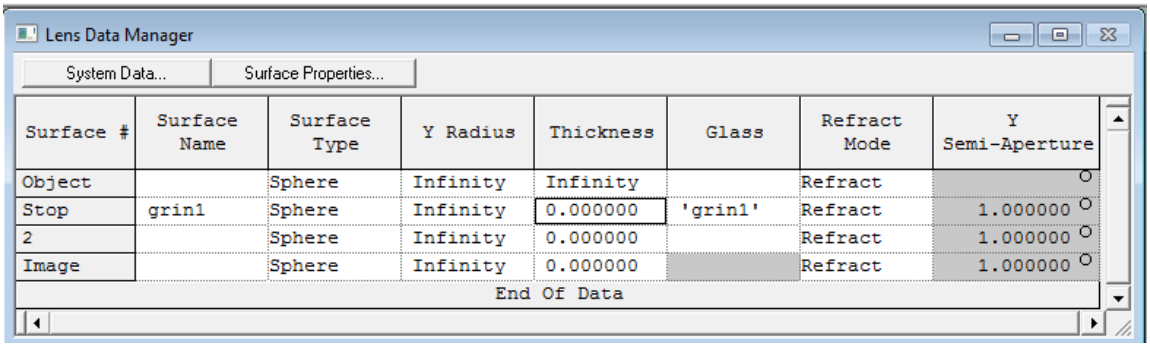
Create private catalog GRIN			Element ID	**1**		step size	**0.05**
			Note: Can't be more than 4 characters				

After all user input fields have been entered, there is a cell that contains a list of commands that create the private catalog glass with the given profile and material properties. This list of commands should be copied and pasted into the Code V command window.

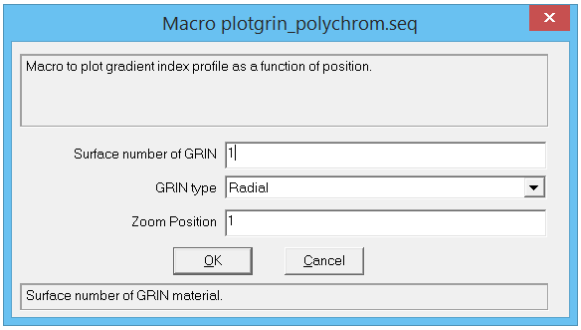
```
PRV; PWL 1300 1200 1100 1000 900 800 700 600 500 400 ; 'grin1' 1.49694317517391 1.50037932335016 1.50296177502131 1.5047988091282
```

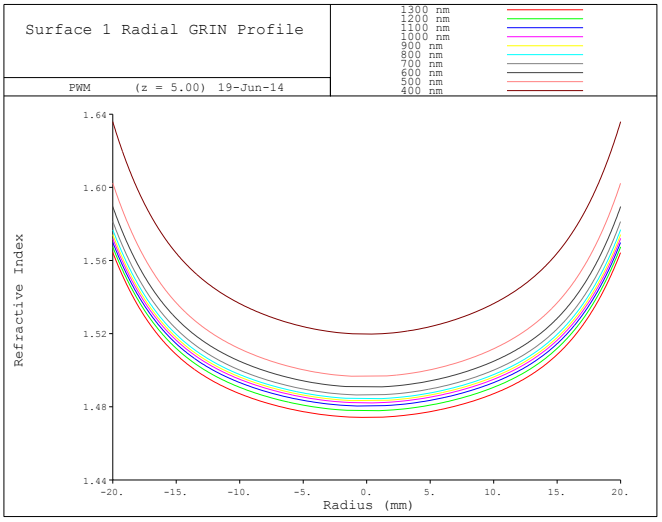


To assign the new GRIN material to one of the surfaces, simply enter the private catalog glass name in the lens data manager. Remember, again, to also label the surface with the same name.



The designer can view the index profile by running the accompanying macro file "plotgrin_polychrom.seq". The user must indicate the surface, profile type, and zoom position.

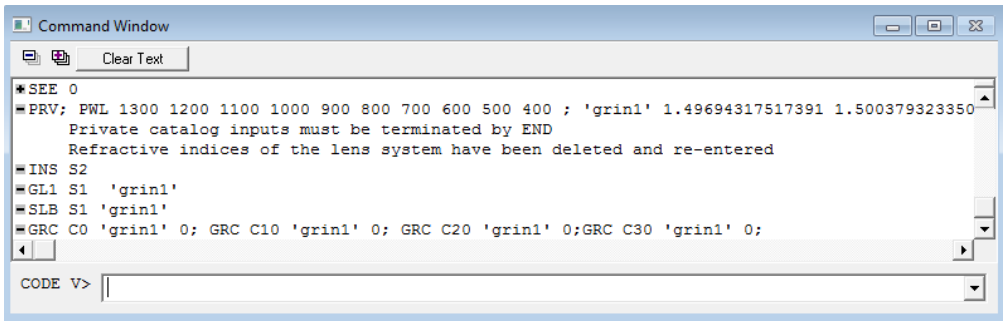




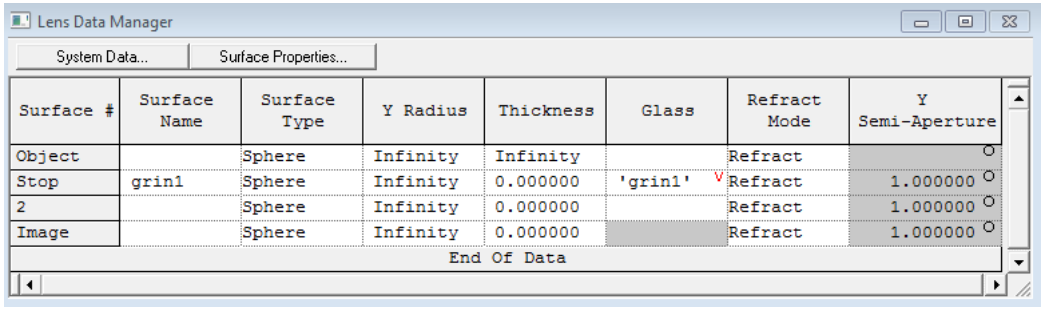
This material with the entered profile can be used in a design just as any other private catalog glass. This is useful for doing polychromatic evaluation of a profile that has been manufactured and measured. The next section covers the basics of optimizing the coefficients of the index polynomial.

III. Optimizing the GRIN Coefficients in Code V

The Excel spreadsheet contains an additional command to vary the GRIN profile coefficients. This should be copied and pasted into the Code V command window. Alternatively, the command can be placed before the AUT command in the optimization sequence file so that it is clear which coefficients are varying for every use of the optimization file.



Now the index profile coefficients at all wavelengths can vary. This can be seen by the small "v" in the lens data manager.



The Excel file creates a list of constraints (over 100 lines) that must be copied and pasted into a .seq file for optimization in Code V.

Creat Optimization Constraints
! CONSTRAINTS FOR OPTIMIZING RADIAL GRADIENT WITH R^2, R^4, and R^6 TERMS ON ELEMENT grin1
! LAST EDITED ON 02-21-2012 BY PETER MCCARTHY
! Define gradient coefficients at all wavelengths
@N0_10_1 == (GRN C0 S'grin1' W1)
@N0_9_1 == (GRN C0 S'grin1' W2)
@N0_8_1 == (GRN C0 S'grin1' W3)
@N0_7_1 == (GRN C0 S'grin1' W4)
@N0_6_1 == (GRN C0 S'grin1' W5)
@N0_5_1 == (GRN C0 S'grin1' W6)
@N0_4_1 == (GRN C0 S'grin1' W7)
@N0_3_1 == (GRN C0 S'grin1' W8)
@N0_2_1 == (GRN C0 S'grin1' W9)
@N0_1_1 == (GRN C0 S'grin1' W10)

This optimization file should also contain any other constraints needed for the designer's system, including focal length, thickness, etc.

CVEdit - [aut_PMMA_PS_radial.seq]

File Edit View Window Help

```
GRC C0 'grin1' 0
GRC C10 'grin1' 0
GRC C20 'grin1' 0
GRC C30 'grin1' 0

AUT
INT Y
DRA S1..I

EFL = 100
CT S'grin1' > 5 < 10
ET S'grin1' > 2

! CONSTRAINTS FOR OPTIMIZING RADIAL GRADIENT WITH R^2, R^4, and R^6 TERMS ON ELEMENT grin1
! LAST EDITED ON 02-21-2012 BY PETER MCCARTHY
! Define gradient coefficients at all wavelengths
@N0_10_1 == (GRN C0 S'grin1' W1)
@N0_9_1 == (GRN C0 S'grin1' W2)
@N0_8_1 == (GRN C0 S'grin1' W3)
@N0_7_1 == (GRN C0 S'grin1' W4)
@N0_6_1 == (GRN C0 S'grin1' W5)
@N0_5_1 == (GRN C0 S'grin1' W6)
@N0_4_1 == (GRN C0 S'grin1' W7)
@N0_3_1 == (GRN C0 S'grin1' W8)
@N0_2_1 == (GRN C0 S'grin1' W9)
@N0_1_1 == (GRN C0 S'grin1' W10)
@N10_10_1 == (GRN C10 S'grin1' W1)
@N10_9_1 == (GRN C10 S'grin1' W2)
@N10_8_1 == (GRN C10 S'grin1' W3)
@N10_7_1 == (GRN C10 S'grin1' W4)
```

Ready

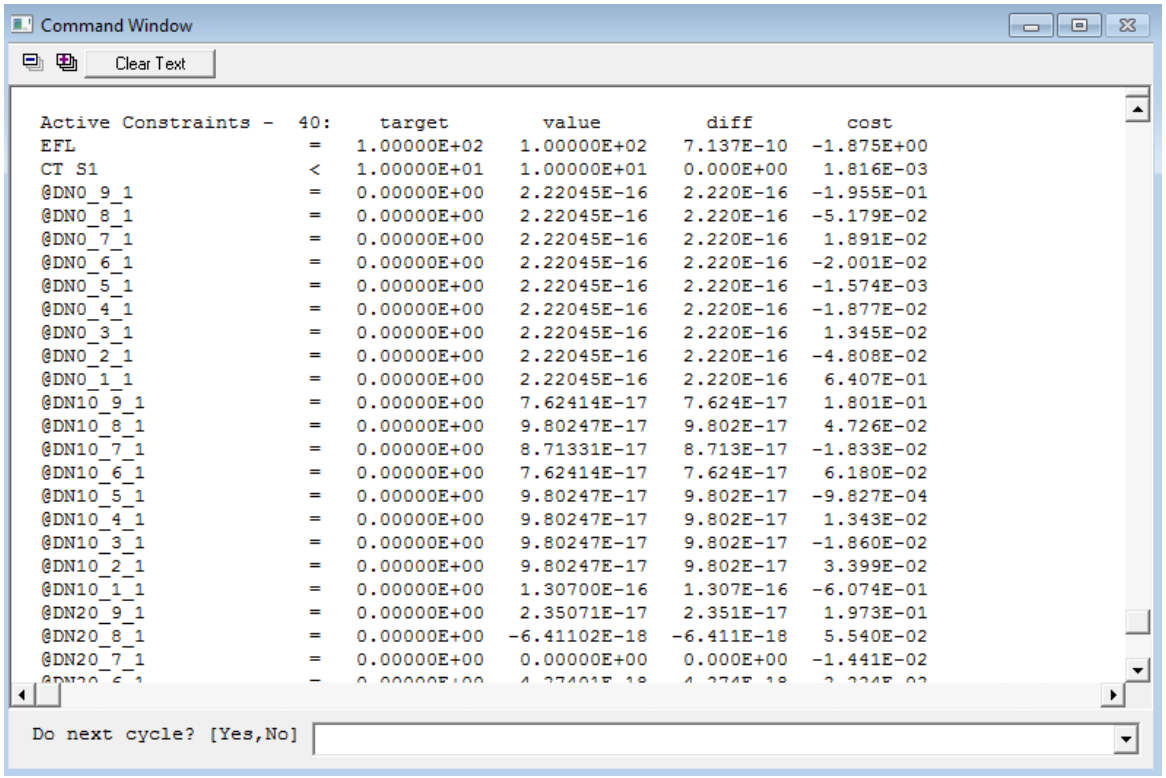
The lens can now be optimized while maintaining the correct dispersion properties. It is essential that the wavelengths used in the lens file are the same as those used when creating the optimization constraints because each user-defined constraint is tied to the material properties at a specific wavelength.

Command Window

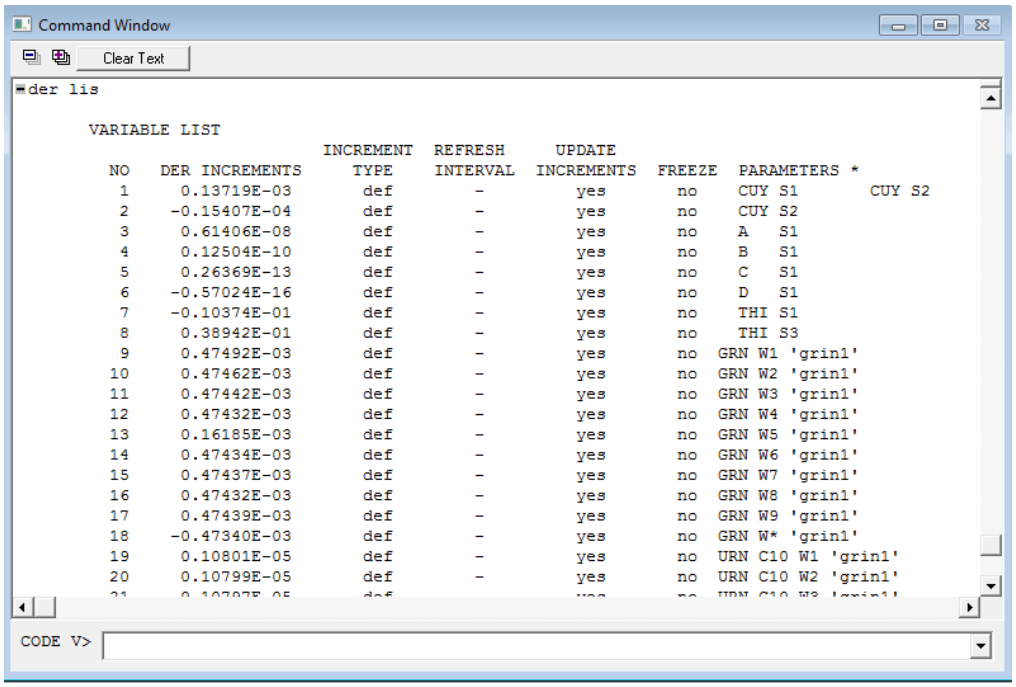
Clear Text

CODE V> in aut1;go

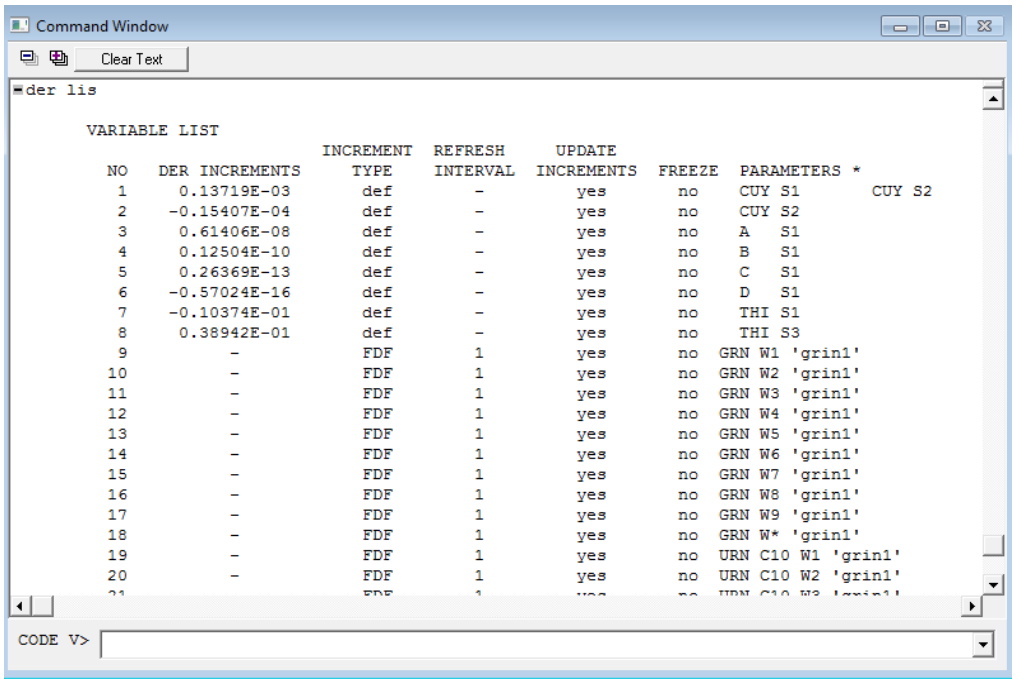
During the optimization process, it is important to monitor the constraints on the GRIN coefficients. Each coefficient is scaled by the appropriate power of the radius so that the value of the user-defined constraint is in Δn space.



In the example above, the GRIN coefficient constraints are all of order 10^{-15} or smaller, which is very well constrained. The designer should be concerned when the magnitudes of these constraints are larger than 10^{-8}. In cases where the constraints are not being obeyed, there are two options the designer can use to gain more control over the optimization. The first is changing the derivative type from default to finite differences, and the second is changing the update interval for the GRIN variable derivative increments. First, it is necessary to attain a list of all variables in the system so the designer can identify which ones correspond to gradient coefficients. This is done with the “DER LIS” command.



In this example, the variables starting at number 9 and going all the way down to number 48 are associated with the gradient. Now, to change the derivative update interval, use the command "DER i9..48 DRC #", where # is how often you want the derivatives recomputed. For example, use "DER i9..48 DRC 1" to update the derivatives after every optimization iteration. Note that the optimization will be noticeably slower when using DRC 1. To change the derivative type to finite differences issue the command "DER i9..48 FDF". Now you can see that the GRIN variables have been changed.



A GRIN element can be optimized inside a more complicated multi-element system by following this same procedure. Additionally, multiple GRIN elements can be optimized in the same lens system, provided they are given different element numbers when creating the constraints. When using multiple gradient materials, it is good practice to comment in the optimization file what the materials are for each GRIN element. These tools will also allow the designer to constrain a GRIN element when using the new Glass Expert feature in Code V. Note that Glass Expert cannot insert a GRIN element into the system, but it will constrain an existing GRIN while it searches for better homogeneous materials in other elements. Early versions of Glass Expert had some issues with changing the user-set derivative properties of the GRIN variables. The designer should make sure that the GRIN constraints are being obeyed.

IV. Quick Start Guide

1. In the "Material Properties" or "Create New Materials" worksheets of the Excel file, choose which material and wavelengths you wish to use.
    a. Use at least 3 wavelengths and as many as 10. Enter a value of zero for any wavelengths you don't wish to use.
    b. List wavelengths from longest to shortest.
2. Copy and paste wavelength, slope, and y-intercept values into the top of the "Radial GRIN Controls" worksheet.
3. Input initial GRIN profile.
    a. Select concentration at r = 0 and the polynomial coefficients at the longest wavelength.
    b. Use the maximum semi-aperture of the lens (rmax) and the plot of the concentration profile to ensure your starting position is within the material bounds.
    c. First-order properties can be monitored.
4. Input optimization controls.
    a. Determine which coefficients you wish to vary during optimization.
    b. Enter upper and lower concentration bounds based on real material properties.
    c. Enter element ID (limited to 4 characters).
    d. Enter ray trace step size.
5. Insert initial GRIN profile into Code V.
    a. Copy the command that starts with "PRV; PWL" and paste it into the Code V command window.
    b. If you wish to optimize the GRIN profile, also copy and paste the command that starts with "GRC".
    c. In the lens data manager, change the material to the correct private catalog glass 'grin*ID*', where *ID* is the (less than 4 character) ID you entered in Excel.
    d. Label the surface with the same name, *grinID*.
6. Copy and paste optimization constraints into a sequence file with any other constraints needed for the lens system.
7. Optimize the system as you normally would by loading in the .seq file.
    a. Use the same wavelengths in Code V as you did when you created the optimization constraints.

b. Monitor the user-defined constraints for the GRIN profile and adjust the derivatives appropriately.
c. View the index profile plot periodically.
d. Change the derivative type to FDF or change the derivative increment to improve optimization behavior.

V. Caveats and Warnings

1. Code V 10.6 made a change to the predefined INDEX function. Code V version 10.5 and earlier are unaffected. From the 10.6 release notes:

   > "The behavior of the INDEX macro function has been changed. The input (X, Y, Z) coordinates now refer to the global reference surface coordinate system defined with the GGS command rather than the local surface coordinates."

   a. The INDEX function is used in the constraints to make sure the index values within the lens element are constrained within the material bounds. It is also used in the plotting macros.
   b. This is only an issue if you change the reference coordinate system for the GRIN to be a surface other than the GRIN surface.
   c. For small shifts of the GGS surface relative to the GRIN surface, the existing constraints might be tolerable.
   d. For a workaround in version 10.6 please contact the UR GRIN group.
2. The refractive index data in the Excel file is either from literature sources or it has been measured at University of Rochester. Verify that the index data is appropriate for the materials you are using.
3. These constraints are based on a linear material model, which may not be the most accurate model for all GRIN materials. Users need to verify that this model adequately describes their materials of interest.